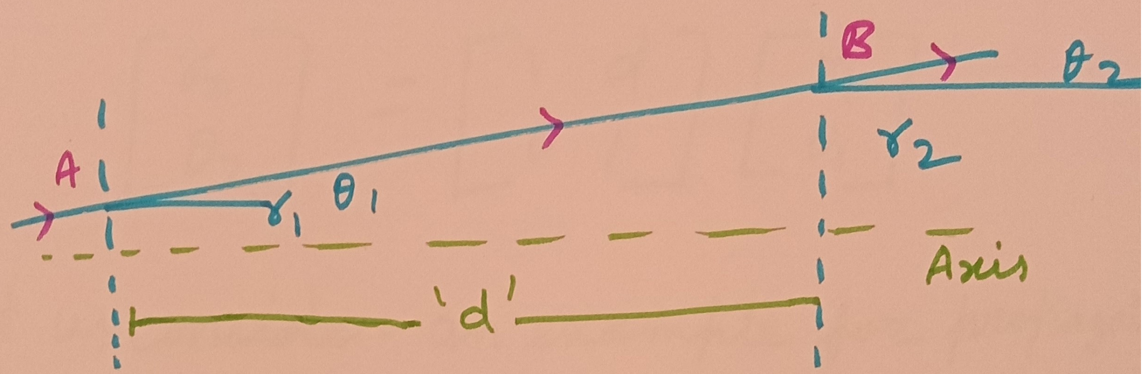
Ray tracing in an optical cavity

Ray Matrix

let us consider paraxial ray travelling with an small angle 'θ' with respect to the optical axis. Since θ is very small

$$\tan\theta \approx \sin\theta \approx \theta$$



The propagating ray can be traced with two parameters 'r' & 'θ'. The distance of ray from the axis is 'r' and its angle with axis is represented by 'θ'.

The ray in the above fig. is determined by 'r_1' & 'θ_1' at 'A' and r_2 & θ_2 at point 'B'. As we can see that this is a simple rectilinear propagtion of light through a distance 'd'.

The output parameters are related to input parameters by the relation

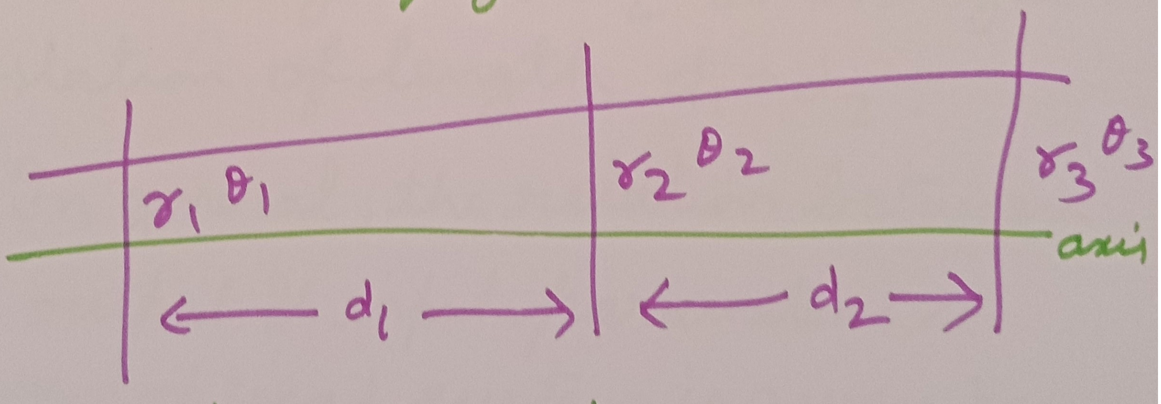
$$r_2 = 1 \cdot r_1 + d \cdot \theta_1$$

$$\theta_2 = 0 \cdot r_1 + 1 \cdot \theta_1$$

Or I can write this in matrix form as

$$\begin{bmatrix} r_2 \\ \theta_2 \end{bmatrix} = \begin{bmatrix} 1 & d \\ 0 & 1 \end{bmatrix} \begin{bmatrix} r_1 \\ \theta_1 \end{bmatrix}$$

let us consider an example for propagation of two free space length d_1 & d_2 as shown in the figure.



As we have learned

$$\begin{bmatrix} r_3 \\ \theta_3 \end{bmatrix} = \begin{bmatrix} 1 & d_2 \\ 1 & 1 \end{bmatrix} \begin{bmatrix} r_2 \\ \theta_2 \end{bmatrix} \quad \&$$

$$\begin{bmatrix} r_2 \\ \theta_2 \end{bmatrix} = \begin{bmatrix} 1 & d_1 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} r_1 \\ \theta_1 \end{bmatrix}$$

or ~~contbr~~ combining two relations

$$\cancel{\begin{matrix} r_3 \\ \theta_3 \end{matrix}} \begin{bmatrix} r_3 \\ \theta_3 \end{bmatrix} = \begin{bmatrix} 1 & d_2 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & d_1 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} r_1 \\ \theta_1 \end{bmatrix}$$

or $$\begin{bmatrix} r_3 \\ \theta_3 \end{bmatrix} = \begin{bmatrix} 1 & d_1 + d_2 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} r_1 \\ \theta_1 \end{bmatrix}$$

Which is obviously correct as it represents the translation of two cascaded length d_1 & d_2 to a single translation of length $d_1 + d_2$.

In general, the relation between the output & input parameters a general optical system is given by ABCD matrix of the form

$$\begin{bmatrix} r_{out} \\ \theta_{out} \end{bmatrix} = \begin{bmatrix} A & B \\ C & D \end{bmatrix} \begin{bmatrix} r_{in} \\ \theta_{in} \end{bmatrix}$$